2.1 Product and Quotient Rules: Quiz A Key

Tutor copy. Every answer was checked by symbolic computation (SymPy, `verify_quiz.py`). All problems are original; provenance is in `PROVENANCE_QA.md`. Prepared with assistance from Claude, an AI model made by Anthropic.

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General grading rules

- **Show work.** A bare correct answer earns at most half credit, except where a row says otherwise.
- **Any valid method.** Simplifying first, the product rule, and the quotient rule are all accepted wherever they apply; no method is required, and long but correct work gets full credit.
- **Value vs. derivative.** A function value where a derivative was asked (or the reverse) earns 0 for that part unless a row says otherwise.
- **Domains.** On Q1, a correct note on where f' does not exist is accepted, never required.
- **Simplifying.** Answers need not be simplified beyond what a row says; equivalent forms are correct.

Quiz A

Item	Answer	Rubric / trap
1(a)	$3x^2\cos x - x^3\sin x$	product rule: $(3x^2)(\cos x) + (x^3)(-\sin x)$. 1 pt rule applied or simplification, 1 pt correct derivative. *Trap:* multiplying the derivatives gives $-3x^2\sin x$; earns 0.
1(b)	$\dfrac{14}{(x+4)^2}$	quotient rule: $\dfrac{3(x+4)-(3x-2)(1)}{(x+4)^2}$. 1 pt rule applied or simplification, 1 pt correct derivative. *Trap:* reversing the numerator gives -14; $-(3x-2)$ needs the minus on both terms.
1(c)	0 (for every x in the domain of f)	$\dfrac{3x(2x^2-1)}{x(2x^2-1)} = 3$ wherever f is defined, a constant. The quotient rule, carried through, also gives 0. 1 pt rule applied or simplification, 1 pt correct derivative. *Trap:* a constant in disguise. The quotient rule is valid but long; its numerator simplifies to 0.
1(d)	$4\sec^2 x$	$\sec x\cos x = 1$ wherever $\sec x$ is defined, so $f(x) = 4\tan x - 1$. The product rule on $\sec x\cos x$ also works: $\sec x\tan x\cos x - \sec x\sin x = \sin x - \sin x = 0$. 1 pt rule applied or simplification, 1 pt correct derivative. *Trap:* a constant hidden in a product.
1(e)	0	$\sec(\pi) = -1$ and $\sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$ are numbers, so $f(x) = -2$ for every x. 1 pt rule applied or simplification, 1 pt correct derivative. *Trap:* no x appears; the quotient rule applied as if $\sec(\pi)$ and $\sin\left(\frac{\pi}{6}\right)$ were functions of x earns 0.
2	$y+\pi^2 = -2\pi(x-\pi)$, i.e. $y = -2\pi x + \pi^2$	1 pt $f(\pi) = \pi^2\cos\pi = -\pi^2$; 2 pts $f'(x) = 2x\cos x - x^2\sin x$; 1 pt $f'(\pi) = -2\pi - 0$; 1 pt equation, any correct form. *Trap:* $\cos\pi = -1$, not 1; $\sin\pi = 0$ removes the second term.
3	$x = \frac{5\pi}{4}$ and $x = \frac{7\pi}{4}$	1 pt $f'(x) = 2 - \csc^2 x$; 2 pts $\sin^2 x = \frac{1}{2}$, $\sin x = \pm\frac{\sqrt{2}}{2}$; 2 pts both solutions in the interval and nothing else. *Trap:* $\sin x = +\frac{\sqrt{2}}{2}$ alone gives $\frac{\pi}{4}$, $\frac{3\pi}{4}$, neither in $(\pi, 2\pi)$; on this interval $\sin x < 0$, so only the negative root counts. Answering "none" earns at most 2.

Item	Answer	Rubric / trap
4(a)	4	Readings: $f(0) = 1$, $f'(0) = -2$, $g(0) = -1$, $g'(0) = 2$ (slopes are rise over run of the segment containing $x = 0$). $p'(0) = f'(0)g(0) + f(0)g'(0)$. 2 pts readings, 1 pt rule, 1 pt value. *Trap:* $f'(0)g'(0) = -4$; $f(0)g(0) = -1$; reading a height where a slope is needed; mixing a value from one graph with a slope from the other.
4(b)	6	Readings: $f(-2) = 3$, $f'(-2) = 0$, $g(-2) = -1$, $g'(-2) = -2$ (slopes are rise over run of the segment containing $x = -2$). $q'(-2) = \dfrac{f'(-2)g(-2) - f(-2)g'(-2)}{[g(-2)]^2}$. 2 pts readings, 1 pt rule, 1 pt value. *Trap:* $f'(-2)/g'(-2) = 0$; reversed numerator gives -6; swapping the solid and dashed graphs gives $-\frac{2}{3}$.
5(a)	2	$h'(x) = 2x\,f(x) + x^2 f'(x)$, $6 = 2(3)(-2) + 9f'(3)$; substitute the given values and solve for $f'(3)$. 2 pts rule written with letters, 1 pt substitution, 1 pt value. *Trap:* the item gives the derivative of the product and asks for a factor's derivative; dropping the $2x\,f(x)$ term gives $\frac{2}{3}$.
5(b)	5	$k'(x) = \dfrac{x\,g'(x) - g(x)\cdot 1}{x^2}$, $1 = \dfrac{2g'(2) - 6}{4}$; substitute the given values and solve for $g'(2)$. 2 pts rule written with letters, 1 pt substitution, 1 pt value. *Trap:* treating k' as $g'(x)/1$ gives 1.
6(a)	$\frac{5}{2}$	$h'(3) = \dfrac{g'(3)f(3) - g(3)f'(3)}{[f(3)]^2}$, with the row $x = 3$. 1 pt rule, 1 pt entries, 1 pt value. *Trap:* h has g on top; the f/g version gives $-\frac{5}{8}$. Columns run x, $g'(x)$, $f(x)$, $g(x)$, $f'(x)$.
6(b)	0	$k(x) = \dfrac{f(2)}{g(1)} = \frac{1}{2}$ for every x, a constant. 1 pt, all or nothing. *Trap:* a quotient of two numbers; running the quotient rule with the $x = 3$ row earns 0.

Points: $10 + 5 + 5 + 8 + 8 + 4 = 40$.